10 Power supply design example part 3 (Frequency response)

A plant model for a digital power supply is used to construct a compensator with enhanced and maximum (fastest) performance. There is no overshoot or oscillation. Both systems are compared to PID-compensated systems. Frequency response is shown.

This example is based upon a digital power supply seminar in which the plant was already known. In part 1, we constructed a maximum-performance compensator. In part 2, we dialed back the performance so that we compare an enhanced performance system with a PID equivalent system. Then we compared the maximum performance system to the PID system. In part 3 we now examine frequency response. The sampling rate of the system is 200 kHz. Frequency response plots are shown for the range 100 Hz to 100 kHz.

The maximum performance compensator (Active Compensation™) is an active design that is given limits on the plant input range and performs run-time computations to assure that these limits are enforced. The compensator may continuously drive (or overdrive) the plant input to the limit for an extended number of cycles, as determined by run-time computations.

This is akin to applying full-throttle for an extended period of time, then after some number of cycles, adjust the throttle as the system approaches setpoint. This is all done while remaining within the prescribed input bounds of the plant and is enforced by run-time computation.

Dialing back the system performance keeps the plant input within bounds without the need for run-time computations to assure that plant input limits are respected. This uses what is called an enhanced-performance compensator. The PID equivalent compensator is then constructed and system performance is compared. Both the enhanced performance and PID compensators are passive compensators constructed at design-time. The filters used in enhanced-performance and PID designs are static and do not change at run-time, whereas the filters used in maximum performance designs may effectively change at runtime depending upon runtime computations.

PID is limited to three numerator coefficients and an integrator. In this example, we will use a matrix solver to find the best-fit PID coefficients for the enhanced performance system we are matching.

Let’s take a moment to clarify terminology and understand system construction arrangements.

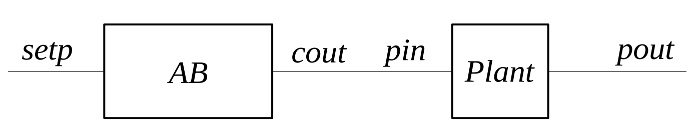

Figure 1 (AB compensator)

Figure 1 shows a compensator AB (Adjustment Block) followed by a Plant model without feedback. The purpose of this arrangement is to shape plant input and output using simulation to confirm system design goals are met, such as overshoot, oscillation, plant input remaining within bounds, plant input sequence, plant output sequence, shape of step response, and so on. This compensator is not deployed in a real physical system.

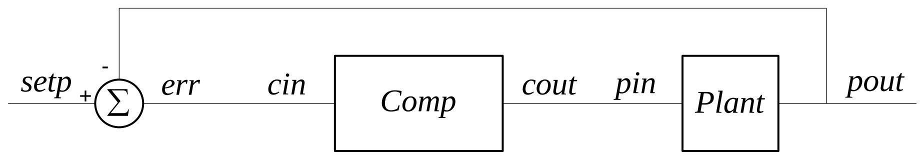

Figure 2 (closed loop compensator)

Figure 2 shows a closed loop compensator (with integrator included) followed by a Plant model. When we are satisfied with system performance using the AB compensator shown in figure 1 above (as determined by system simulation), the closed loop compensator is directly computed from the AB block. In simulation, the same setpoint (aka reference) sequence applied in figure 1 or figure 2 will produce the identical plant output sequence $pout$. This arrangement is deployed in a real physical system.

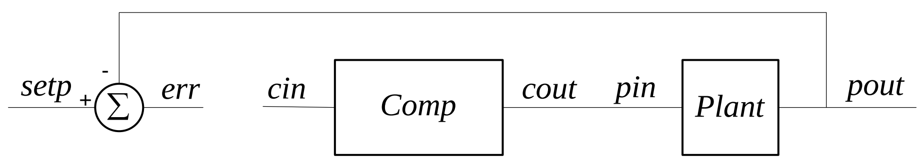

Figure 3 (closed loop compensator operating open loop)

Figure 3 shows the closed loop compensator open at the compensator input cin. In all other respects, this is the same arrangement that is shown in figure 2 (the compensator and plant is identical). The purpose of this arrangement is to obtain a bode plot of open loop system operation with the frequency sweep applied directly to the compensator input cin, and system response observed at the plant output $pout$ using either the plant model or the real physical plant. From the bode plot, the general appearance of the response, the crossover frequency f_x, the gain margin, and the phase margin can all be observed. Alternate methods, such as a frequency response analyzer can also be used, as appropriate.

To determine closed-loop bandwidth, the arrangement shown in figure 2 is used. For open-loop frequency response, the arrangement shown in figure 3 is used.

“Impulse Response” is terminology used in the context of continuous-time systems, whereas “Unit Response” is terminology used in the context of discrete-time systems. In every situation in which “Impulse Response” is used in the context of discussing discrete-time systems, it is intended to convey the meaning of “Unit Response.”

The system used in the seminar was

$$G_p(s) = \frac{3.8968*10^{-5}s + 11.139}{7.5091*10^{-10}s^2 + 1.8337*10^{-5}s + 1}$$

and the proposed compensator was

$$G_c(s) = 1.60974*10^{-5} * \frac{s^2 + 1.34540*10^4 s + 4.04259*10^8}{s}$$

In this example, we are normalizing the plant to be $0 \le pout \le 1$ and restricting the range of the plant input to be $0 \le pin \le 1$.

First, let's use Matlab to show the system, compensator, and open loop frequency response of the seminar plant and compensator.

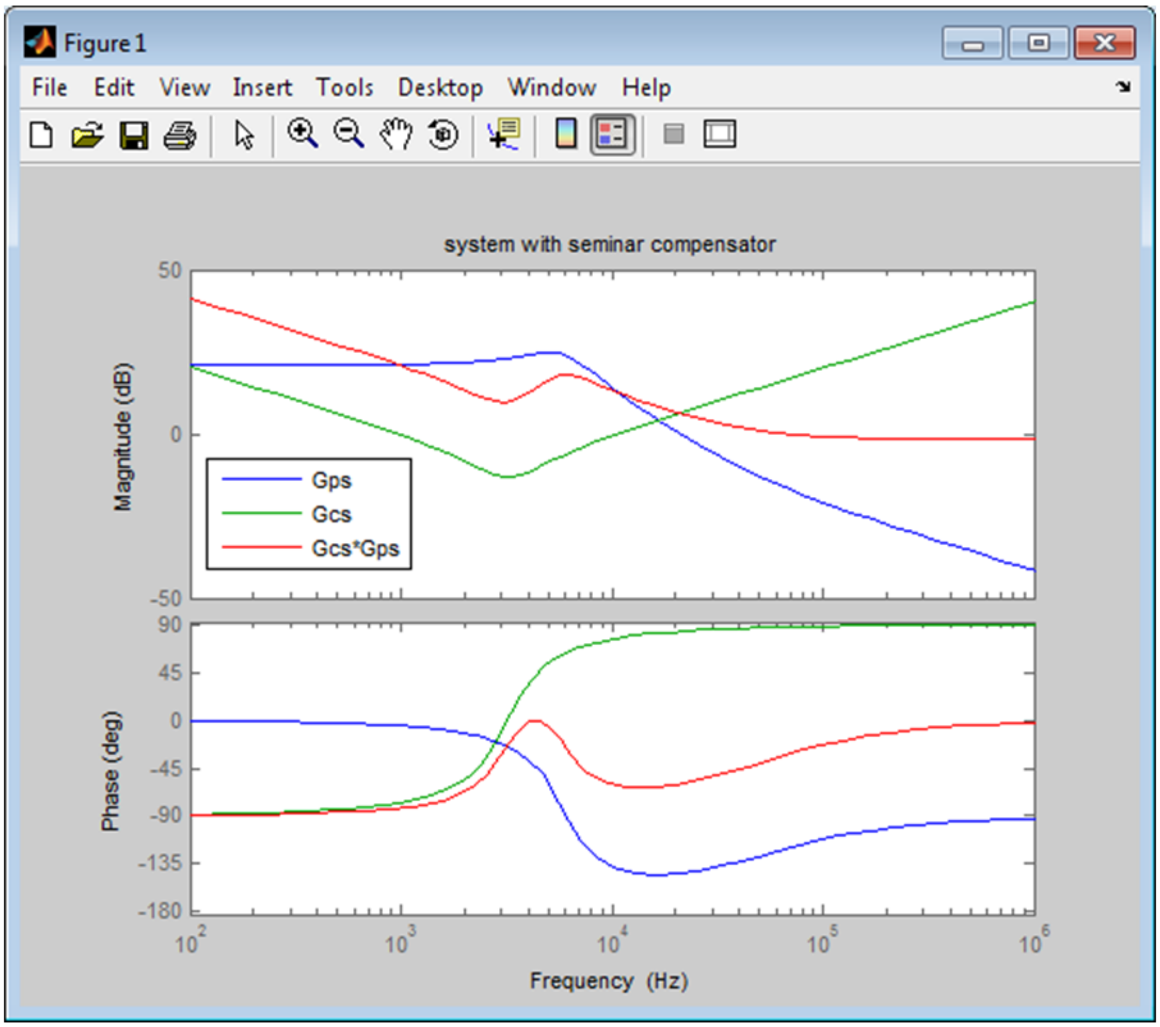

Figure 4

Figure 4 shows the seminar plant, the proposed seminar compensator, and the open loop frequency response.

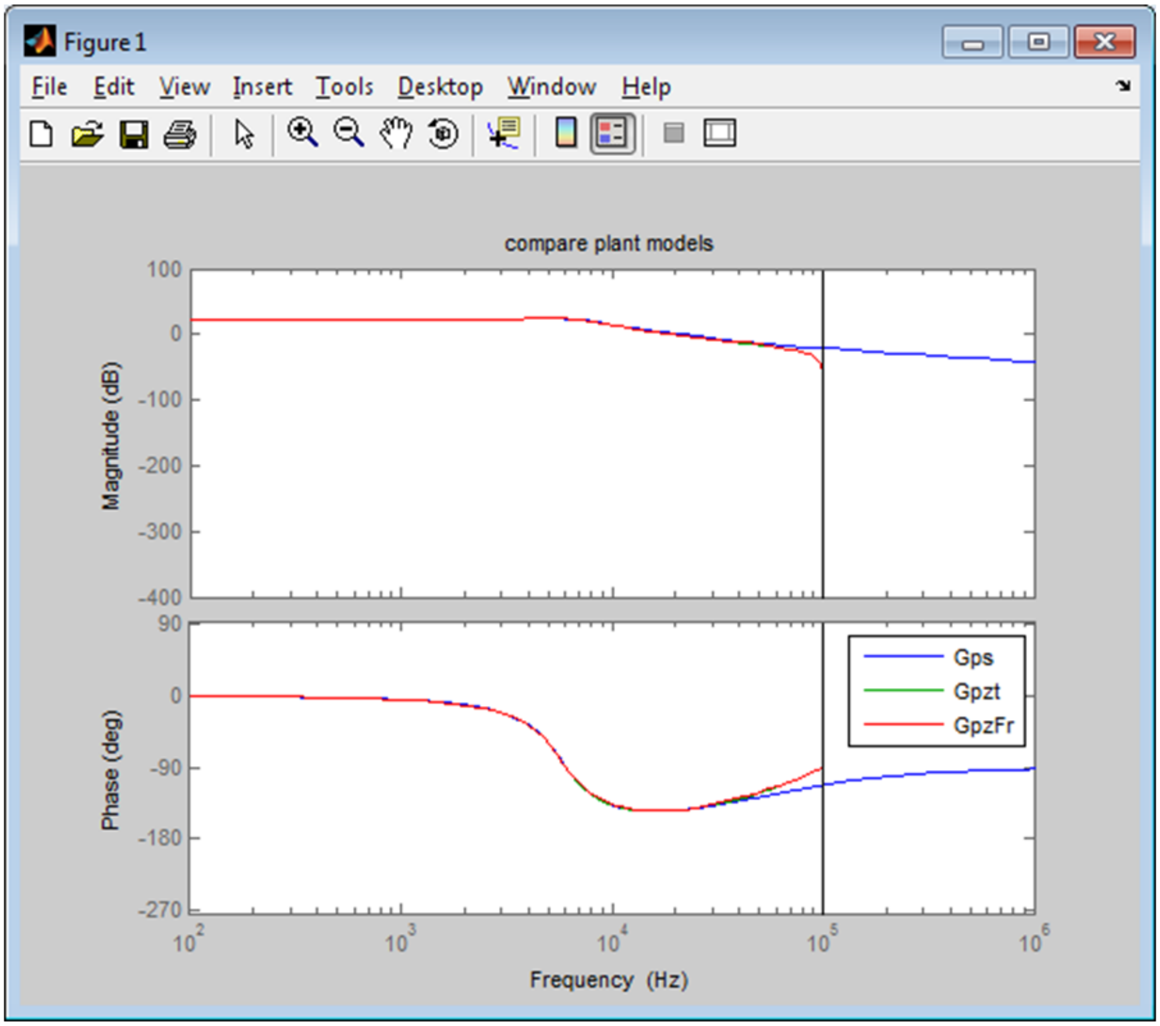

Figure 5

Figure 5 compares the frequency response of the uncompensated plant $G_p(s)$, and two discrete-time approximations $G_{pt}(z)$, and $G_{pFr}(z)$. The first approximation $G_{pt}(z)$ is the bilinear (Tustin) transform, and the second approximation $G_{pFr}(z)$ is our equivalent transform which we will be using for frequency response evaluation in this paper. The point of showing them together is to demonstrate equivalency as each approximation overlays the other.

Let's start with the original (normalized) plant model:

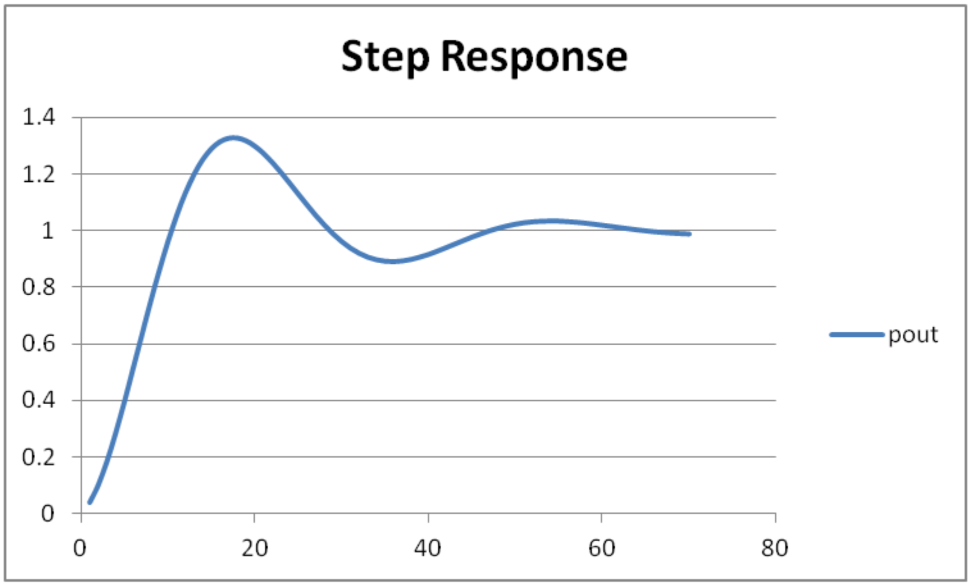

Figure 6

Figure 6 shows the plant's uncompensated Step Response. There is 33% overshoot and oscillation.

Let's compute the Impulse Response.

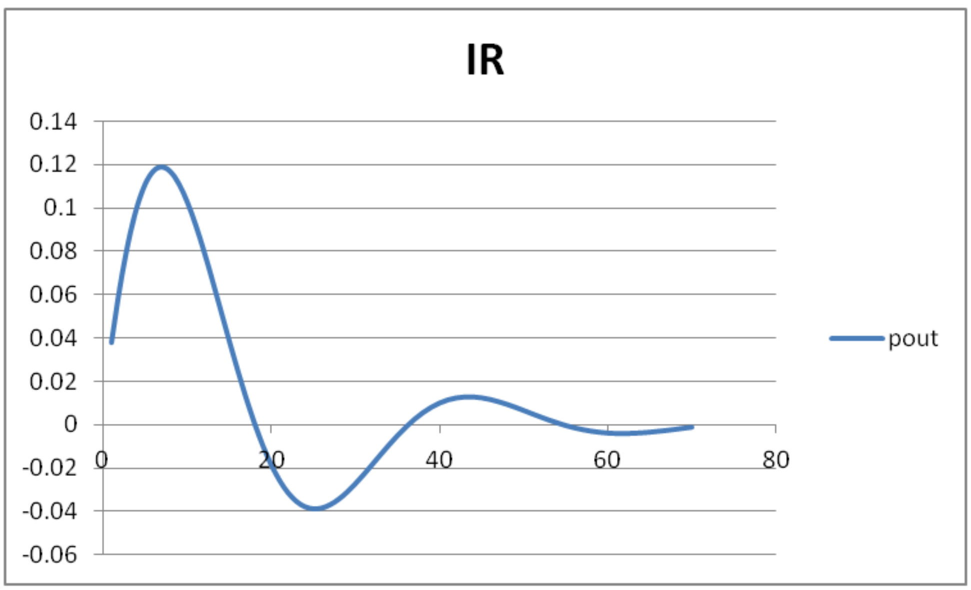

Figure 7

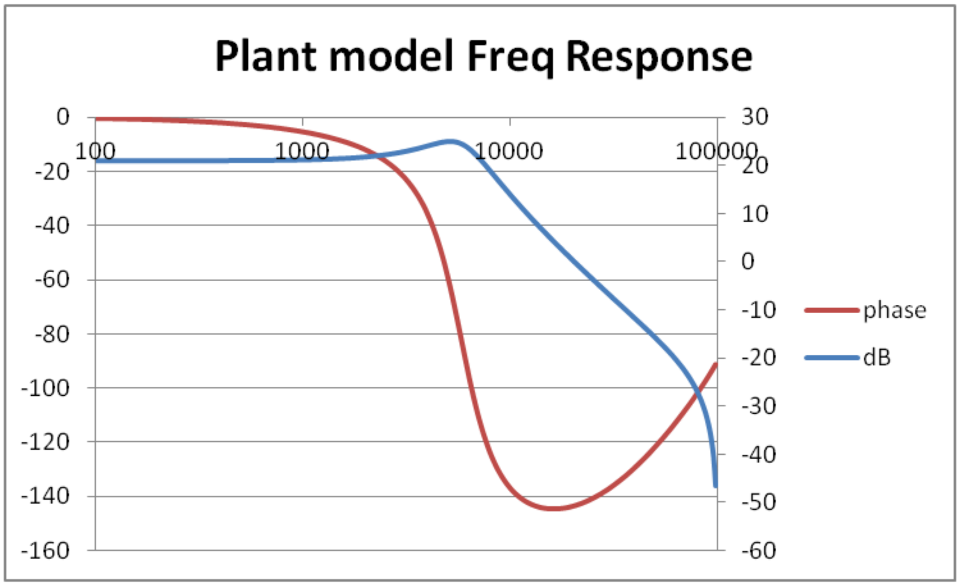

Figure 8

Figure 8 shows the frequency response of the uncompensated unnormalized plant. Compared to the plot in Figure 5, the scaling is different, but the information is the same.

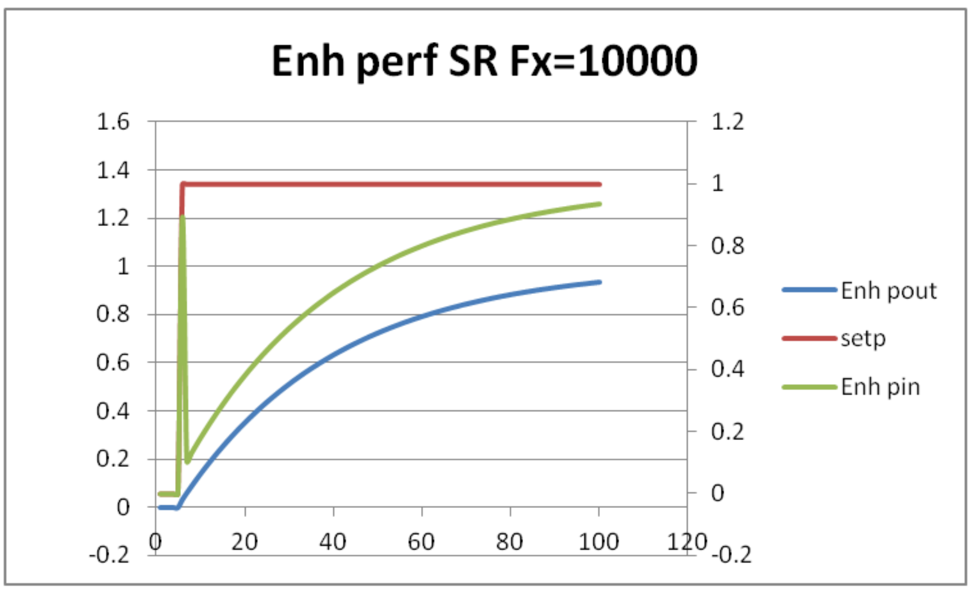

Figure 9

Figure 9 shows the enhanced performance system with a step input.

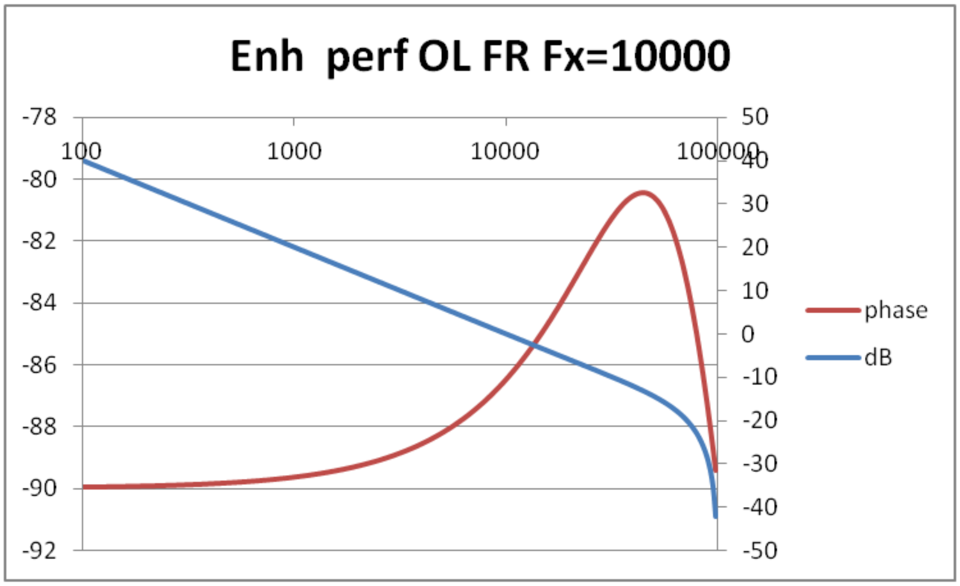

Figure 10

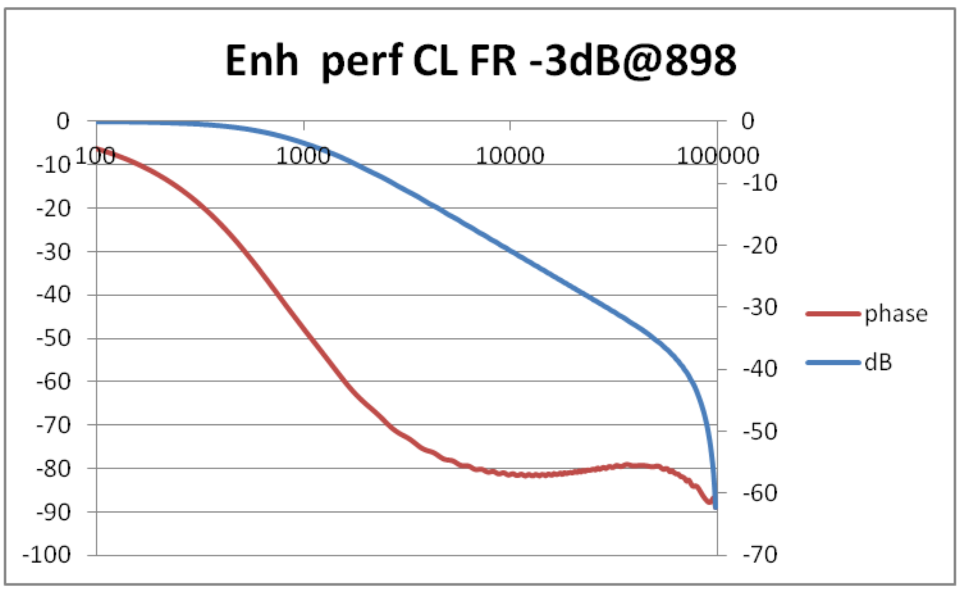

Figure 11

Figure 10 shows the open loop frequency response with a crossover frequency of 10000 Hz. Figure 11 shows the closed-loop bandwidth of the enhanced performance system to be 898 Hz.

The enhanced performance system was evaluated with the plant input range restricted to ($0 \le pin \le 1$). This is intentional so that an equivalent PID system can be constructed directly from the enhanced performance compensator and evaluated.

After constructing the equivalent PID system, we get the following simulation:

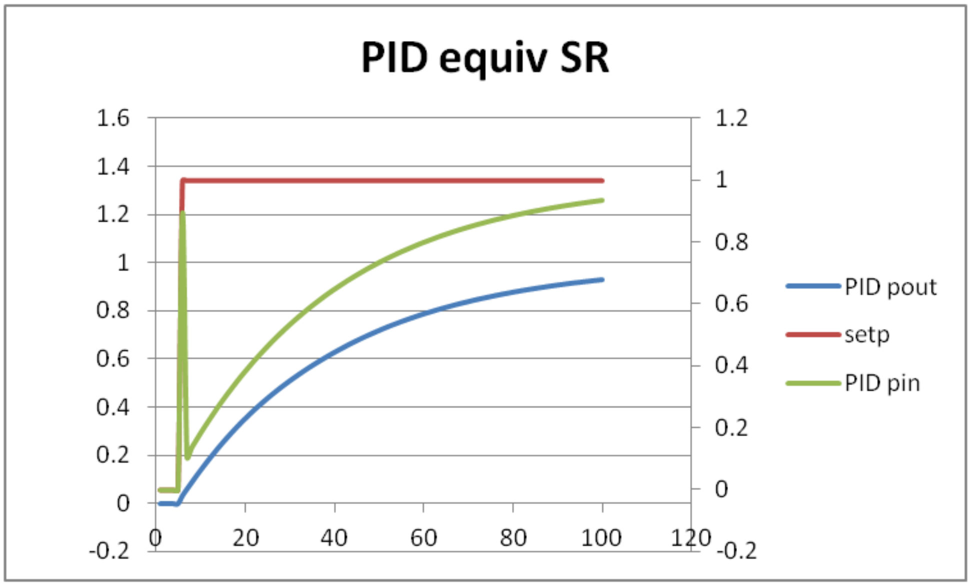

Figure 12

Figure 12 shows the equivalent PID system with a step input. The PID system appears to be identical to the enhanced performance system, but there is small maximum error of 0.45%. While not an absolutely perfect match to enhanced performance, it's very, very close.

Note that we computed the PID coefficients directly from the enhanced performance compensator. Ziegler–Nichols tuning, or any other tuning method is not required – the least-squares best-fit computed result works as-is without adjustment.

Now, let's have a look at the PID frequency response:

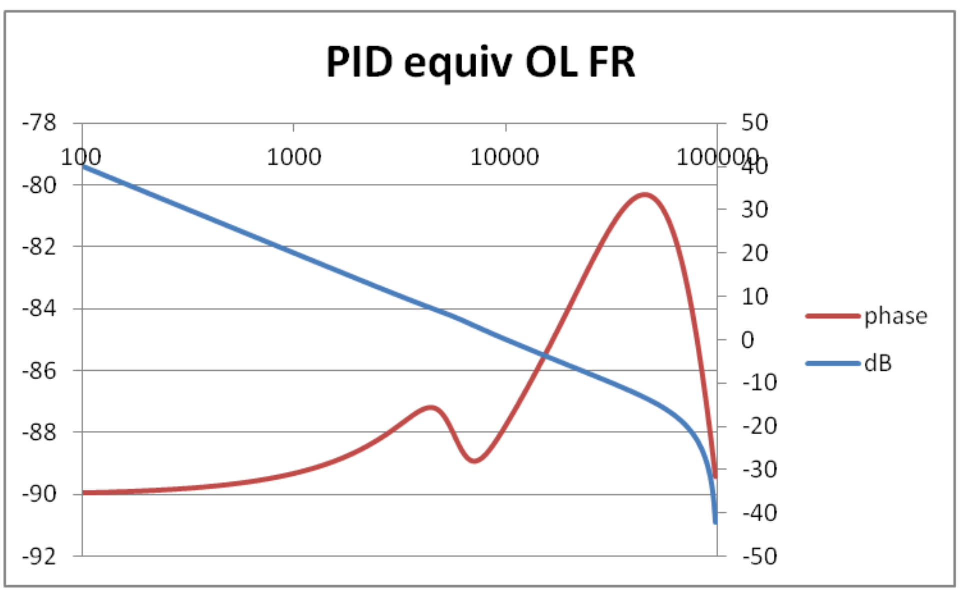

Figure 13

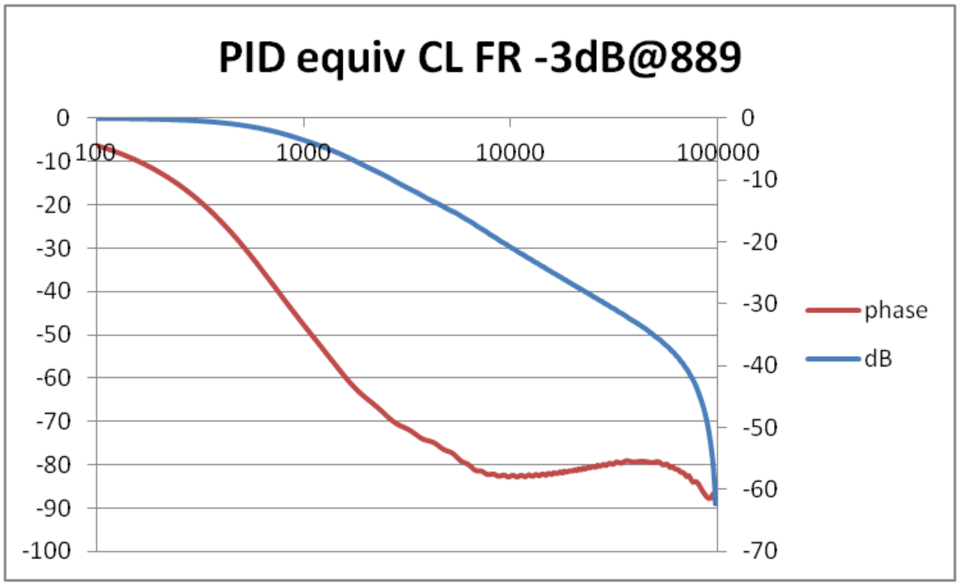

Figure 14

Figure 13 shows the open loop frequency response of the PID equivalent system. Figure 14 shows the closed-loop bandwidth of the PID equivalent system to be 889 Hz.

Next, we evaluate the maximum performance system:

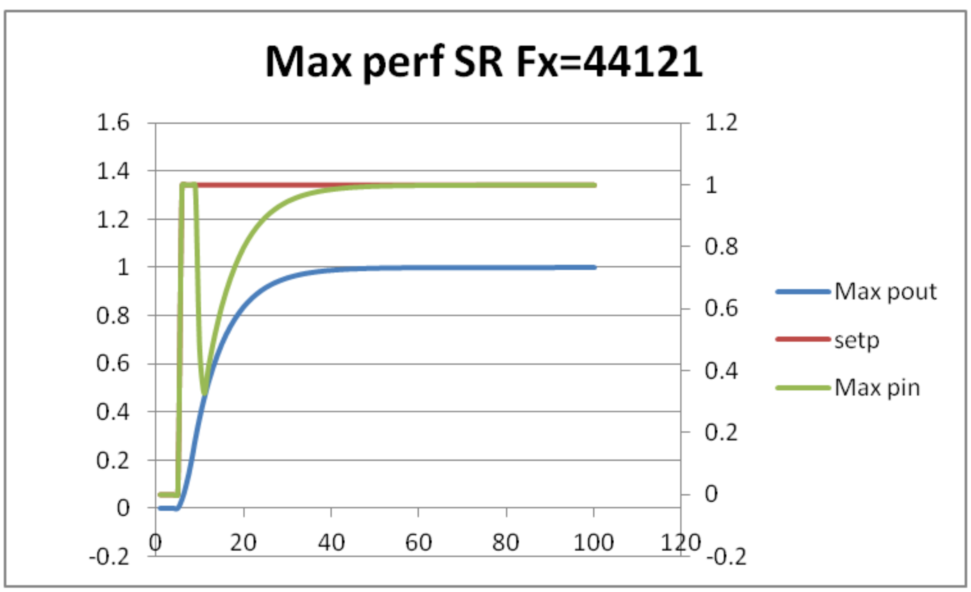

Figure 15

Figure 15 shows the maximum performance system with a step input. The plant input range is still restricted to ($0 \le pin \le 1$) so we can see the plant input limiting while the system is operating.

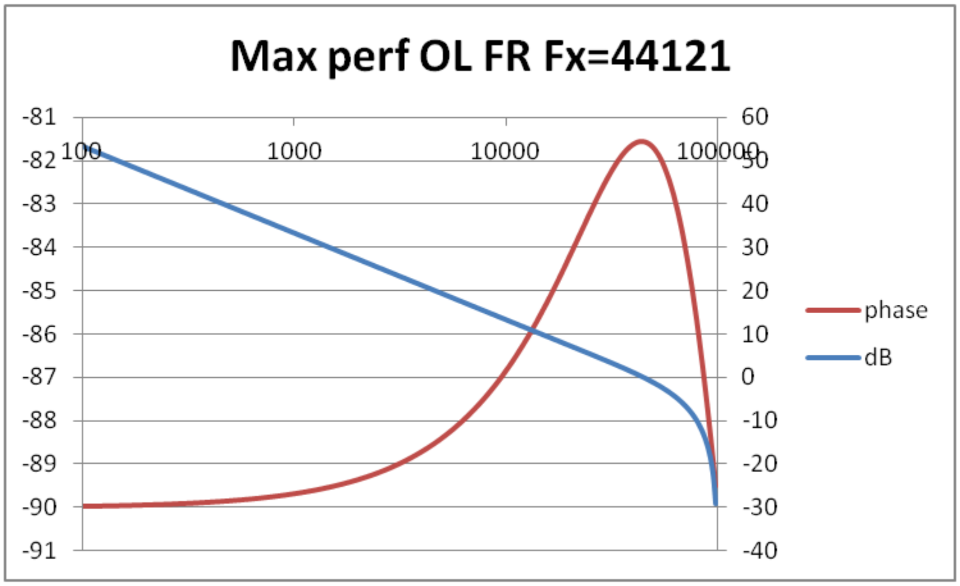

Figure 16

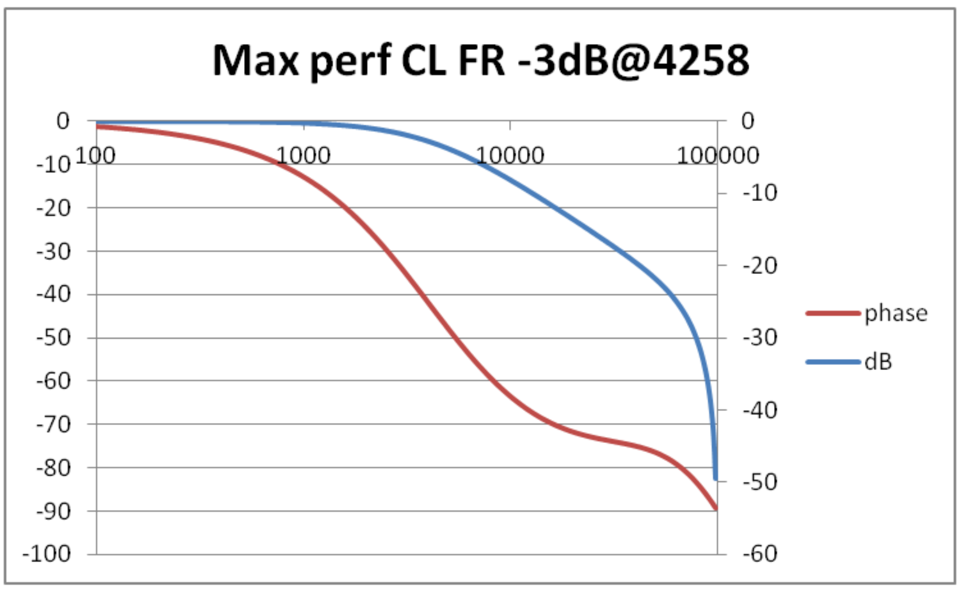

Figure 17

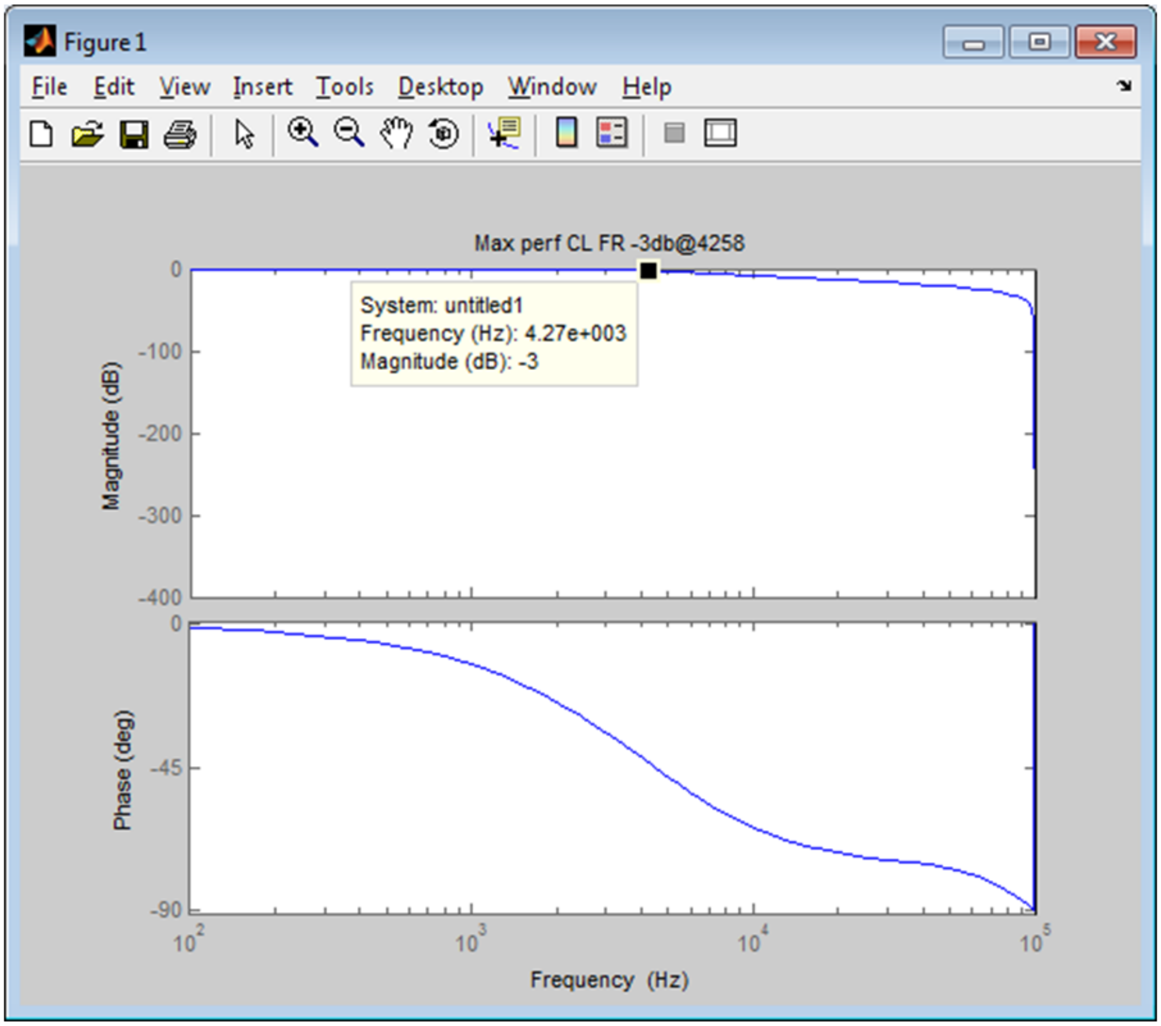

Figure 18

Figure 16 shows the open loop frequency response with a crossover frequency of 44121 Hz. The phase starts at -90 and remains within the range -90 <= phase < -81.

Figure 17 shows the closed loop bandwidth of the maximum performance system to be 4258 Hz.

Figure 18 shows the Matlab bode plot. The scaling is different, but the information in the plot is the same.

Going back to figure 16, we see a healthy open loop frequency response which is taken as a reasonable bound on maximum possible performance that is available. Various intermediate levels of performance are available as plant input restrictions are relaxed or the limits on the systems range of operation are tightened.

The maximum performance outlined in Figures 16 and 17 above is just one instance of a wide spectrum of possible solutions. The system depicted here was intentionally chosen to be very aggressive as a demonstration of the capability of this method.

System	dB @ Freq
Enhanced performance	-3.01 dB @ 898 Hz
PID	-3.01 dB @ 889 Hz
Maximum performance (err <= 1.00)	-3.01 dB @ 1008 Hz
Maximum performance (err <= 0.50)	-3.01 dB @ 2056 Hz
Maximum performance (err <= 0.33)	-3.01 dB @ 3137 Hz
Maximum performance (err <= 0.25)	-3.01 dB @ 4258 Hz

Figure 19

Figure 19 shows the bandwidth summary for the various normalized system configurations and should be interpreted as a sliding scale of performance, that is, as the error term shrinks dynamically at runtime, the performance increases dynamically at runtime. This implies that for a small enough range of error input, the performance will approach maximum, while respecting prescribed plant input limits.

As an example, suppose we decide that the plant input limits should be $0 \le pin \le 1$. We should expect 1008 Hz as a minimum for any error term sequence. If the error term stays below 0.50, we can expect 2056 Hz as a minimum; if the error term stays below 0.33, we can expect 3137 Hz as a minimum, and so on, all while respecting the plant input limits of $0 \le pin \le 1$.

Summary:

This method of compensator construction starts with a derived plant model already in hand, and creates a compensator that drives (or overdrives) the plant to the defined limit for an extended number of cycles to achieve maximum performance.

We used a matrix solver to compute a least-squares best-fit PID compensator coefficients directly from the enhanced performance compensator. The PID compensated system was compared to the enhanced performance compensated system and found that the performance was very nearly identical.

The computed PID coefficients did not require Ziegler–Nichols tuning, or any other tuning method – the least-squares best-fit computed result worked as-is without adjustment.

We found that the step response and closed loop frequency response plots were nearly identical for the enhanced performance system compared to the PID system. Maximum performance frequency response compared to the PID and enhanced performance frequency response was improved by nearly 5x as the error term became smaller.