

# 9 Power supply design example part 2 (PID comparison)

© Rick Gros, August 4, 2019

---

A plant model for a digital power supply is used to construct a compensator with enhanced and maximum (fastest) performance. There is no overshoot or oscillation. Both systems are compared to PID-compensated systems with and without system delay.

---

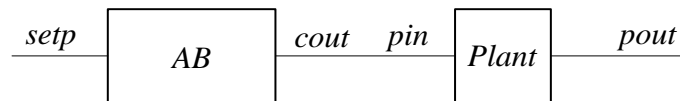
This example is based upon a digital power supply seminar in which the plant is already known. In part 1, we constructed a maximum-performance compensator. Now in part 2, we dial back the performance so that we compare an enhanced performance system with a PID equivalent system.

The maximum performance (Active Compensation™) compensator is an active design that is given limits on the plant input range and performs run-time computations to assure that these limits are enforced. The compensator may continuously drive (or overdrive) the plant input to the limit for an extended number of cycles, as determined by run-time computations.

Dialing back the system performance keeps the plant input within bounds without the need for run-time computations to assure that plant input limits are respected. This uses what is called an enhanced-performance compensator. The PID equivalent compensator is then constructed and system performance is compared.

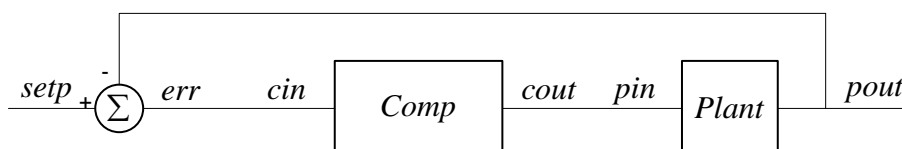
PID is limited to three numerator coefficients and an integrator. In this example, we will use a matrix solver to find the best-fit PID coefficients to match the enhanced performance system.

Let's take a moment to clarify terminology and understand system construction arrangements.



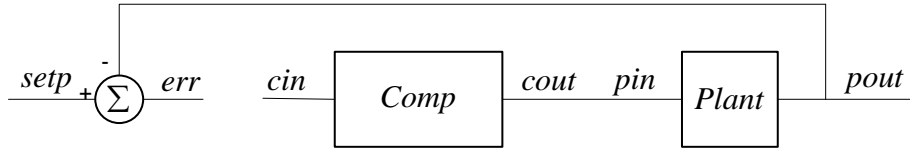
**Figure 1 (AB compensator)**

Figure 1 shows a compensator *AB* (Adjustment Block) followed by a Plant model without feedback. The purpose of this arrangement is to shape plant input and output using simulation to confirm system design goals are met, such as overshoot, oscillation, plant input remaining within bounds, plant input sequence, plant output sequence, shape of step response, and so on. This compensator is not deployed in a real physical system.



### Figure 2 (closed loop compensator)

Figure 2 shows a closed loop compensator (with integrator included) followed by a Plant model. When we are satisfied with system performance using the  $AB$  compensator shown in figure 1 above (as determined by system simulation), the closed loop compensator is constructed directly from the  $AB$  block. In simulation, the same setpoint (aka reference) sequence applied in figure 1 or figure 2 will produce the identical plant output sequence  $pout$ . This arrangement is deployed in a real physical system.



### Figure 3 (closed loop compensator operating open loop)

Figure 3 shows the closed loop compensator open at the compensator input  $cin$ . In all other respects, this is the same arrangement that is shown in figure 2 (the compensator and plant is identical). The purpose of this arrangement is to obtain a bode plot of open loop system operation with the frequency sweep applied directly to the compensator input  $cin$ , and system response observed at the plant output  $pout$  using either the plant model or the real physical plant. From the bode plot, the general appearance of the response, the crossover frequency  $f_x$ , the gain margin, and the phase margin can all be observed. Alternate methods, such as a frequency response analyzer can also be used, as appropriate. The bode plot is discussed in a future paper.

“Impulse Response” is terminology used in the context of continuous-time systems, whereas “Unit Response” is terminology used in the context of discrete-time systems. In every situation in which “Impulse Response” is used in the context of discussing discrete-time systems, it is intended to convey the meaning of “Unit Response.”

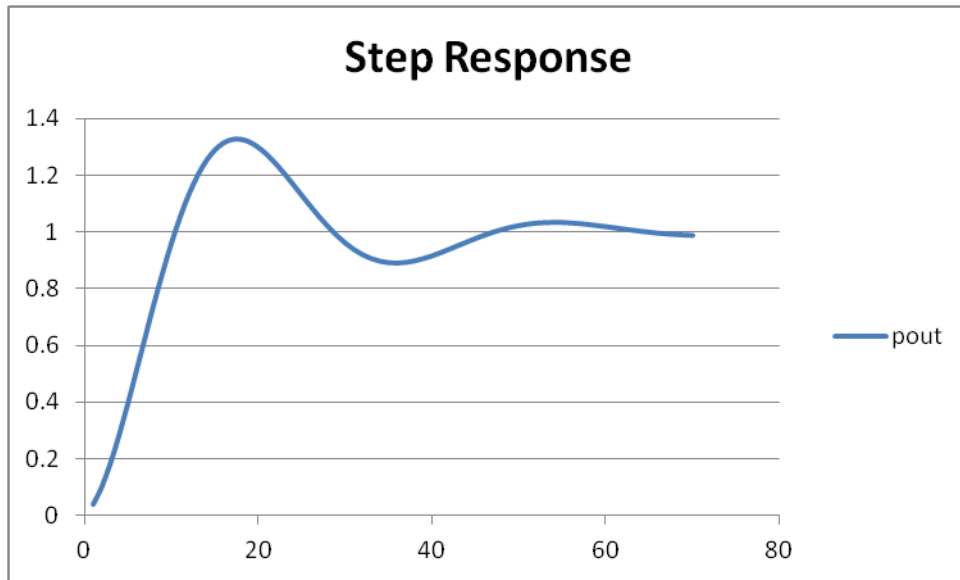
The system used in the seminar was

$$G_p(s) = \frac{3.8968 \cdot 10^{-5} s + 11.139}{7.5091 \cdot 10^{-10} s^2 + 1.8337 \cdot 10^{-5} s + 1}$$

and the proposed compensator was

$$G_c(s) = 1.60974 \cdot 10^{-5} \cdot \frac{s^2 + 1.34540 \cdot 10^4 s + 4.04259 \cdot 10^8}{s}$$

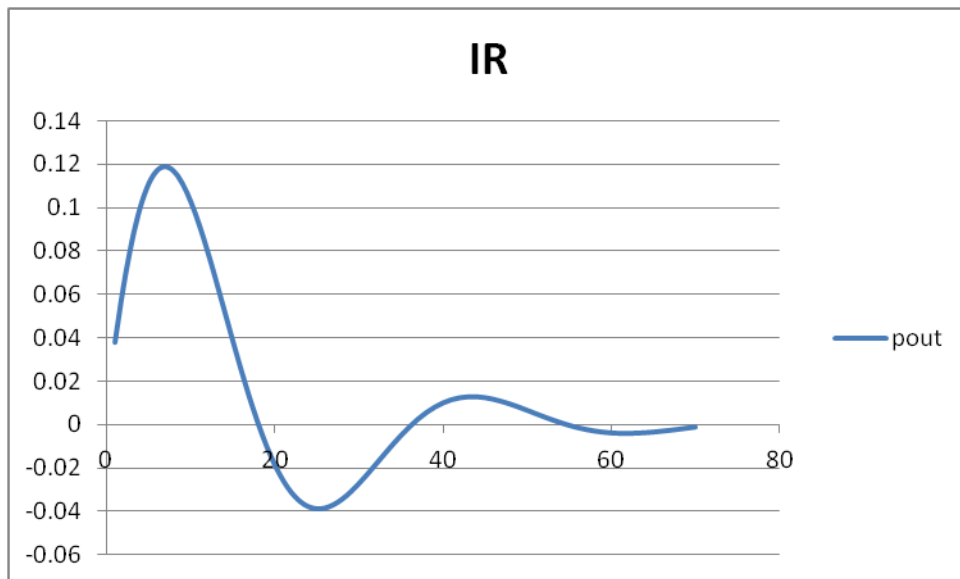
In this example, we are normalizing the plant to be  $0 \leq pout \leq 1$  and restricting the range of the plant input to be  $0 \leq pin \leq 1$ . Let's start with the original (normalized) plant model:



**Figure 4**

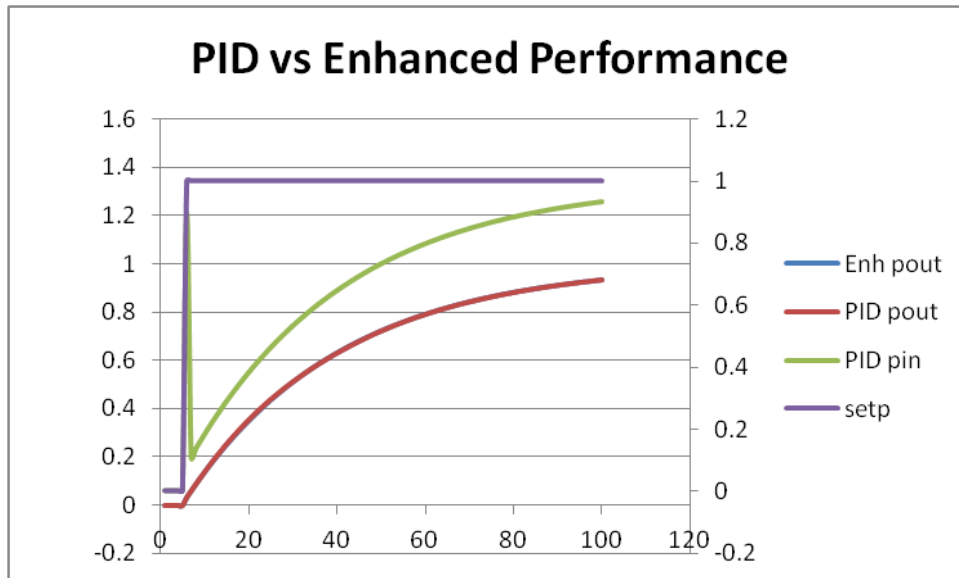
Figure 4 shows the plant's Step Response. There is oscillation and 33% overshoot.

Let's compute the Impulse Response:



**Figure 5**

Let's compare an enhanced performance compensator to a PID compensator. We use a matrix solver to compute a least-squares best-fit PID compensator and see how it performs. In this example, we are restricting the range of the plant input to be:  $0 \leq pin \leq 1$ .

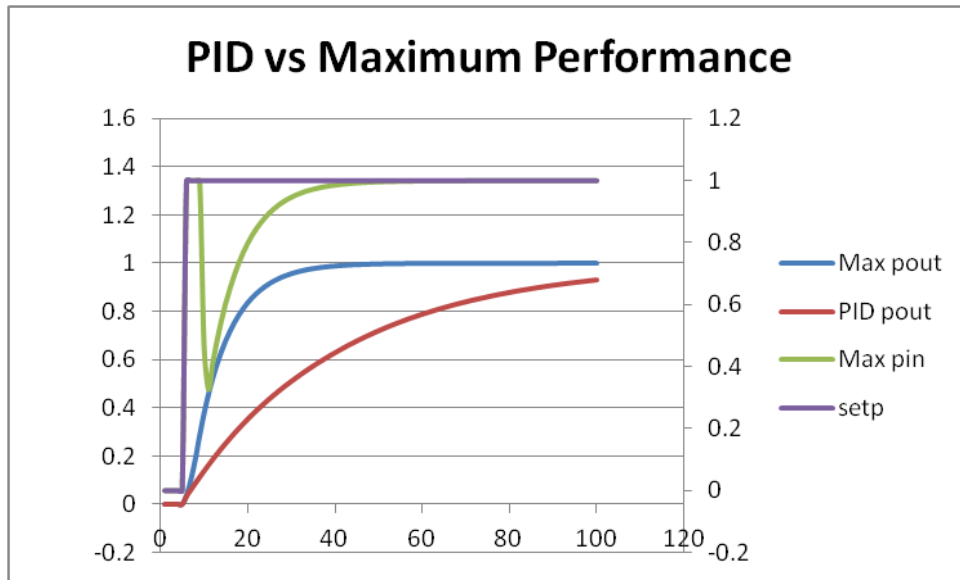


**Figure 6**

Figure 6 shows the plant output in a closed-loop system using an enhanced performance compensator. This is the benchmark against which we compare the plant output in a closed-loop system using a PID compensator. Also shown is the setpoint (a step) and the plant input in the PID system, which we can see remains in bounds ( $0 \leq pin \leq 1$ ).

In this situation, the PID compensator-based system output almost perfectly overlays the enhanced-performance compensator-based system output. It may appear to perfectly overlay, but there is small maximum error of 0.31%. While not an absolutely perfect match to enhanced performance, it's very, very close.

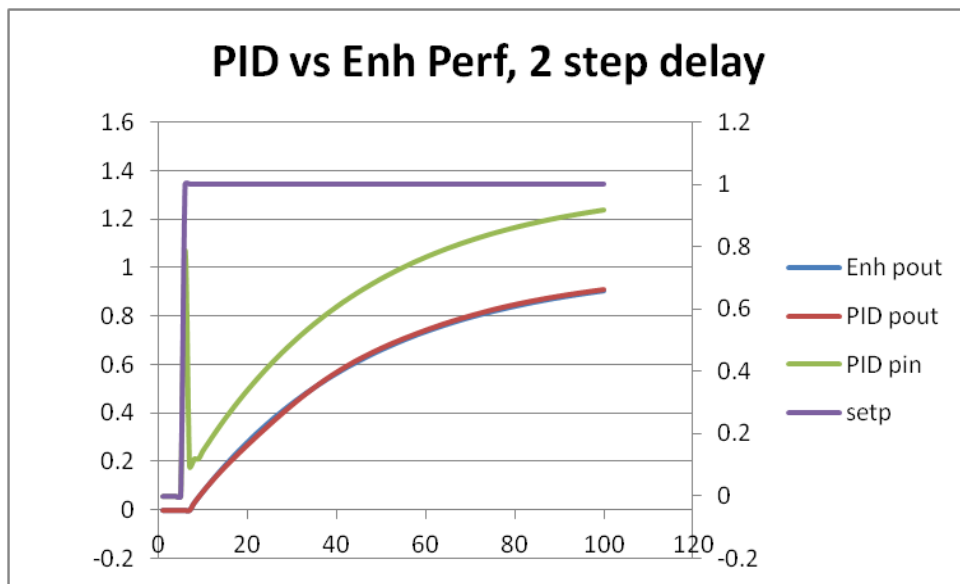
Note that we computed the PID coefficients directly from the enhanced performance compensator. Ziegler–Nichols tuning, or any other tuning method is not required – the least-squares best-fit computed result works as-is without adjustment.



**Figure 7**

Figure 7 shows a maximum performance compensator (Active Compensation™) and plant. Compared to the PID solution, the maximum performance solution arrives on-setpoint much faster.

Next, let's deliberately introduce 2 steps of delay from control effort to plant output. Again, we use a matrix solver to compute a least-squares best-fit PID compensator, but this time, we deliberately include the 2 steps of delay in the plant.

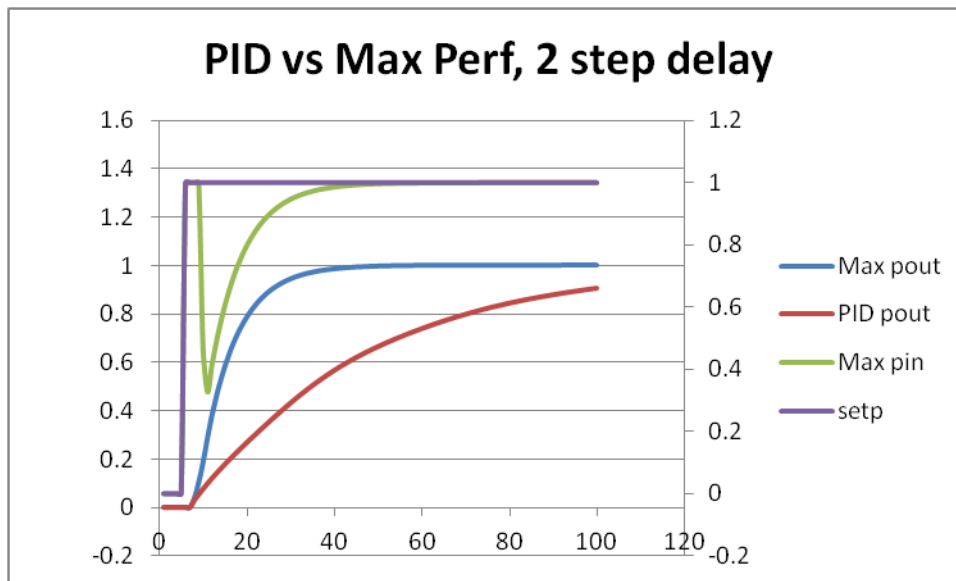


**Figure 8**

Figure 8 shows the plant output in a closed-loop system using an enhanced performance compensator and 2 steps of delay in the plant. This is the benchmark against which we compare the plant output in a closed-loop system using a PID compensator. Also shown is the setpoint (a unit step) and the plant input in the PID system.

With delay in the plant, the PID compensator-based system output is somewhat degraded, compared to the enhanced-performance compensator-based system output. Deviation from the reference is now 1.2% worst case.

Let's have another look at the maximum performance compensator, with 2 steps of delay in the plant.



**Figure 9**

Figure 9 shows a maximum performance compensator and plant. Compared to the PID solution, the maximum performance solution arrives on-setpoint much faster, and the 2 steps of plant delay are properly managed.

Comparing the run-time computational requirements for a plant with 2 steps of delay, for PID we have 4 multiply-accumulates. Enhanced and max performance require 7 multiply-accumulates.

## Summary:

This method of compensator construction starts with a derived plant model already in hand, and creates a compensator that drives (or overdrives) the plant to the defined limit for an extended number of cycles to achieve maximum performance.

A matrix solver was used to compute a least-squares best-fit PID compensator directly from the enhanced performance compensator. The closed-loop PID compensated system was compared to the

closed-loop enhanced performance compensated system and found that the performance was very nearly identical.

The computed PID coefficients did not require Ziegler–Nichols tuning, or any other tuning method – the least-squares best-fit computed result worked as-is without adjustment.

Two steps of delay was intentionally introduced into the plant, and a new PID compensator was constructed. Other than a delay of 2 steps, the enhanced performance system output was identical with or without the delay. The PID compensated system was degraded by a worst-case error of 1.2%.

Other than a delay of 2 steps, the maximum performance system output was identical with or without the delay. Both enhanced and maximum performance compensated systems properly managed delay in the plant without overshoot or oscillation.

Comparing the run-time computational requirements, for PID we have 4 multiply-accumulates, whereas enhanced and max performance require 7 multiply-accumulates (could be optimized to 5 in a system without delay). So, for a modest increase in run-time computational effort, dramatic improvement in system performance can be achieved.